Sample Question

MATHEMATICS

CMA-104: SEMESTER-II

(Real Analysis)

$$\frac{\textit{Full Marks}: 80}{\textit{Pass Marks}: 35}$$

Time: 3 hours

The figures in the margin indicate full marks for the questions

1. Choose and rewrite the correct answer for each of the following questions: 1×3=3

(a) Which of the following is an open set?

(i) $(0,1)\cup[-1,0)$

(ii) $[0,1)\cup(-1,0]$

(iii) $(0,1]\cup[-1,0)$

(iv) $(0,1]\cup(-1,0)$

(b) For the sequence $\{u_n\}$ where $u_n = \begin{cases} 3, & \text{if } n=3k \\ -5, & \text{if } n=3k-1 \\ 4, & \text{if } n=3k-2 \end{cases}$, $k \geq 1$. The value of the $\underline{\lim}\, u_n$ and $\overline{\lim}\, u_n$ are respectively

(i) 3 and 4

(ii) 4 and -5

(iii) -5 and 4

(iv) 4 and 3

(c) A positive term series $\sum \frac{1}{n^p}$ is convergent if and only if

(i) $p > 1$

(ii) $p = 1$

(iii) $p < 1$

(iv) $p \geq 1$

2. Write very short answer for each of the following questions: 1×6=6

(a) State the order completeness property in **R**.

(b) Define limit point of a given set S of real numbers.

(c) Is arbitrary intersection of open sets necessarily open? Give an example.

(d) Find the set of limit points of the sequence

$$\{a_n\} = \text{Sin}\frac{n\pi}{3}, n \in N.$$

(e) "Every bounded sequence has a limit point" the converse of the statement is not always true. Give an example.

(f) Define conditionally convergent series.

3. Write short answer for each of the following questions: 3×5=15

(a) Find the infimum and supremum of the set $\left\{\frac{(-1)^n}{n}, n \in N\right\}$. Which of these belongs to the set?

(b) Show that every open interval is an open set.

(c) Show that the sequence $\{S_n\}$, where

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}, n \in N$$

cannot converges.

(d) Show that the series $\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + ...$ is convergent.

(e) Prove that every absolutely convergent series is convergent.

4. Write answer for each of the following questions: 4×5=20

(a) Prove that intersection of arbitrary collection of closed set is a closed set but (by suitable example) arbitrary union of closed set need not be closed.

(b) Define subsequence of a sequence. Show that every subsequence of a convergent sequence converges to the same limit.

(c) Show that the sequence $\{u_n\}$, where

$$u_n = \frac{2}{n+n} + \frac{2}{n+(n-1)} + ... + \frac{2}{n+2} + \frac{2}{n+1}, n \in N$$

is convergent.

(d) Show that the series $1 + r + r^2 + r^3 + ...$ (i) converges if $r < 1$ and (ii) diverges if $r \geq 1$.

(e) Discuss the nature of the series

$$\sum \frac{(n+1)^n x^n}{x^{n+1}}.$$

5. (a) State and prove Bolzano Weierstrass theorem for set. 6

(b) Prove that every open cover of the closed and bounded interval [a, b] admits of a finite subcover. 6

Or

Prove that a set is closed if and only if its complement is open. 6

6. (a) Show that a necessary and sufficient condition for the convergence of a sequence $\{S_n\}$ is that, for each $\in > 0$ there exists a positive integer m such that

$$\left|S_{n+p} - S_n\right| < \in, \ \forall n \geq m \text{ and } p \geq 1.$$ 6

(b) If $\{S_n\}$ is a sequence, such that

$$S_{n+1} = \sqrt{\frac{ab^2 + S_n^2}{a+1}}, b > a, \ \forall n \geq 1 \text{ and } S_1 = a > 0$$

then show that the sequence $\{S_n\}$ is an increasing bounded above sequence and $\lim_{n\to\infty} S_n = b$. 6

Or

State and prove nested interval theorem. 6

7. (a) State and prove Leibnitz test for alternating series. 6

(b) Test for convergence the series $\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+...$ 6

Or

Examine the convergence of the series $\frac{x}{1+x}+\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}+...$ 6

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